

On the interaction between a geothermal borehole and groundwater flows

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ABSTRACT

The presence of aquifers can highly affect the heat exchange between geothermal boreholes and the ground. To optimally design them, theoretical models for the thermal interaction of geothermal boreholes with groundwater flows are required. The present work exploits the presence of large disparities in time and length scales, using matched asymptotic expansion techniques, to build a mathematically rigorous and physically-sound model that accounts for the presence of creeping groundwater flows. The derived model not only exhibits great performance compared to detailed numerical simulations but also serves to critically assess the merits and limits of the state of the art.

INTRODUCTION

Due to ongoing climate change, mankind has embarked in a large scale decarbonization of all human activities. Since heating and cooling of buildings is one of the main contributors to world energy consumption, with a share of almost 23% (International Energy Agency 2021), the harnessing of low enthalpy geothermal energy to develop energy-efficient heating, ventilation, and air conditioning (HVAC) systems is considered a key action against climate change. A geothermal HVAC system consists in a water-to-water heat pump connected to a geothermal heat exchanger comprised of vertical boreholes. As shown in Figure 1, each of these boreholes is equipped with one or more coaxial or U-shaped probes through which a heat carrying liquid flows to exchange heat with the ground.

The correct sizing of geothermal heat exchangers ensures the prescribed efficiency for the HVAC system is satisfied over the whole lifetime of the building, typically 100 years. This requires long-term predictions of the thermal response of the geothermal heat exchanger. Unfortunately, detailed numerical simulations of the whole problem are unfeasible for engineering purposes (Chiasson et al. 2000; Lou et al. 2021). Hence, simplified theoretical models are used instead which are accurate, flexible, and fast.

All theoretical models take into account conduction of heat in the ground (Cui et al. 2024; Li and Lai 2015). For rocks and soils of low permeability it is the dominant heat transfer mechanism so that the accuracy of predictions in such grounds is high. However, the convective transport of heat due to groundwater flows is often equally relevant, especially in fractured igneous and metamorphic rocks and in gravelly soils (Chiasson et al. 2000; Rico and Hermanns 2020). For those grounds, only models that incorporate this second heat transfer mechanism are able to correctly forecast the thermal response of the geothermal heat exchanger.

The goal of the present work is the mathematically rigorous derivation of a physically-sound theoretical model that accounts for the presence of creeping groundwater flows and seamlessly integrates into the coherent theoretical framework

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Figure 1 Sketch of a typical geothermal borehole.

pursued by the second author since 2011 (Hermanns 2020, 2021). Additionally, thanks to its mathematically rigorous derivation, the presented model also serves to critically assess the theoretical and conceptual merits and limits of the state of the art. This discussion along with all details of the presented model can be found in Rico and Hermanns 2024a, 2024b.

SCALE ANALYSIS OF THE PROBLEM

Figure 1 shows the sketch of a typical geothermal borehole that consists in a vertical borehole of depth H and radius r_b into which several pipes are placed forming one or more coaxial or U-shaped probes. The heat carrying liquid flows with a bulk velocity V along these pipes to exchange heat with the surrounding ground. The space between pipes and ground is usually filled up with grout to promote the aforementioned heat exchange and to avoid the cross-contamination of aquifers.

Three characteristic times can be constructed out of the previous parameters, namely, the characteristic residence time $t_r \sim H/V$ of the heat carrying liquid in the pipes, the characteristic transversal diffusion time $t_b \sim r_b^2/\alpha_g$, where α_g is the effective thermal diffusivity of the ground, and the characteristic longitudinal diffusion time $t_H \sim H^2/\alpha_g$. Computing these characteristic times using real-world values for H, r_b, V , and α_g reveals that t_r is of order minutes, t_b is of order hours, and t_H is of order centuries (Hermanns and Pérez 2014).

The heat injection/extraction imposed by the HVAC system onto the geothermal heat exchanger introduces a fourth characteristic time, namely, t_q . Since the heating and cooling needs of a building vary on an hourly, daily, weekly, monthly, and yearly basis, the characteristic time t_q presents a large spectrum of values that go from minutes up to decades (Hermanns and Pérez 2014). The present work focuses on the most relevant operating conditions for which t_q is much larger than t_b but much smaller than t_H .

The aim of the present work is to incorporate the effect of groundwater flows in the thermal response of geothermal boreholes. Hence, a fifth characteristic time emerges, namely, the characteristic residence time $t_c \sim r_b/U_{\infty}$ of the

groundwater stream in the vicinity of the borehole, where U_{∞} is the effective seepage velocity of the groundwater flow (Rico and Hermanns 2020). The ratio of t_c to the characteristic transversal diffusion time t_b is of the order of the Peclet number of the groundwater flow which, in the present work, is considered to be small compared to unity:

$$Pe = \frac{r_b U_\infty}{\alpha_g} \sim \frac{t_b}{t_c} \ll 1 \qquad \text{so that} \qquad t_r \ll t_b \ll t_c \ll t_q \ll t_H. \tag{1}$$

This disparity in time scales will be exploited to derive approximate, albeit accurate, solutions to the thermal interaction of geothermal boreholes with creeping groundwater flows.

Negligible axial heat transfer

The heat transfer problem in the ground can be described through independent two-dimensional problems formulated in planes perpendicular to the borehole (Hermanns and Pérez 2014; Rico and Hermanns 2024b). This is possible thanks to two features of the problem. First, heat conduction along the borehole is negligible compared to heat conduction in the radial direction as a consequence of t_b and t_q being small compared to t_H . Second, aquifers mostly flow perpendicular to the borehole due to the small slopes of piezometric pressure levels in the ground.

FORMULATION OF THE PROBLEM

The borehole wall is impermeable so no heat convection takes place in the grout filling up the borehole. Consequently, heat transfer inside the borehole is solely governed by heat conduction. Nonetheless, the presence of aquifers introduces heat convection as a second heat transfer mechanism in the ground. To address this problem the porous medium approach is used to avoid formulating and solving the heat transfer problem in the intricate voids of the ground (Nield and Bejan 2017). Additionally, local thermal equilibrium between soil and groundwater is assumed so that a single unsteady convection-diffusion energy conservation equation governs the thermal response of the ground (Nield and Bejan 2017). The resulting governing equations in grout and ground are

$$\frac{\partial T}{\partial t} = \alpha_b \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} \right] \qquad \text{and} \qquad \frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} = \alpha_g \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} \right], \quad (2)$$

respectively. When writing these equations, constant values for the thermal diffusivity of the grout α_b and for the effective thermal diffusivity of the ground α_g are assumed. This simplification is justified by the small temperature variations expected in the problem, at most of 10°C-20°C, which lead to negligible variations in the involved thermal characteristics. The effective groundwater velocity field (v_r , v_θ) is obtained from solving in an exact way the fluid mechanical problem in the ground (Rico and Hermanns 2020, 2024b).

The solution to the formulated governing equations must also satisfy the following continuity conditions in temperature and normal heat flux at the borehole wall in which k_g is the effective thermal conductivity of ground and k_b is the thermal conductivity of grout:

$$T\Big|_{r=r_b^-} = T\Big|_{r=r_b^+}$$
 and $-k_b \frac{\partial T}{\partial r}\Big|_{r=r_b^-} = -k_g \frac{\partial T}{\partial r}\Big|_{r=r_b^+}.$ (3)

The sought solution must also fulfill the following boundary condition at the outer surface of each pipe j located within the borehole:

$$-k_b r_{pj} \frac{\partial T}{\partial r_j}\Big|_{r_j = r_{pj}} = \frac{T_j(t) - T\Big|_{r_j = r_{pj}}}{R_{pj}},\tag{4}$$

where R_{pj} represents the inner thermal resistance of pipe *j*. It encompasses all heat transfer phenomena between the heat-carrying liquid at $T_j(t)$ and the outer pipe's wall at $T|_{r_j=r_{pj}}$. That is, the turbulent transport of heat inside the fluid and the quasi-steady heat conduction occurring through the pipe's wall (Hermanns and Pérez 2014).

The bulk temperature $T_j(t)$ of the fluid in pipe *j* is conveniently set to ensure the prescribed heat injection rate per unit pipe length $q_j(t)$ is satisfied at all times:

$$T_j(t) = \frac{R_{pj}}{2\pi} q_j(t) + \frac{1}{2\pi} \int_{-\pi}^{\pi} T\Big|_{r_j = r_{pj}} \mathrm{d}\theta_j \qquad \text{with} \qquad q_j(t) = \int_{-\pi}^{\pi} -k_b \frac{\partial T}{\partial r_j} \Big|_{r_j = r_{pj}} r_{pj} \,\mathrm{d}\theta_j. \tag{5}$$

Finally, the solution to the formulated problem must tend to the unperturbed ground temperature T_{∞} far away from the borehole and at the beginning of the problem:

$$r \to \infty : T \to T_{\infty}$$
 and $t = 0 : T = T_{\infty}$. (6)

ASYMPTOTIC SOLUTION TO THE PROBLEM

The heat transfer problem to solve is comprised of two governing equations, Eq. (2), continuity conditions at the borehole wall, Eq. (3), boundary conditions at the pipe walls, Eqs. (4) and (5), and a boundary condition far from the borehole, Eq. (6). In the absence of an exact solution, the present work derives an approximate, albeit accurate, one using matched asymptotic expansion techniques (Lagerstrom 1988). These techniques exploit the presence of large disparities in time and length scales to decompose mathematically complex problems into simpler ones. To facilitate the identification of these disparities, an order of magnitude estimation of the three groups of terms comprising the governing equation in the ground is performed:

$$\underbrace{\frac{\partial T}{\partial t}}_{\sim \frac{\Delta T}{t_q}} + \underbrace{v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta}}_{\sim \frac{\Delta T}{t_c}} = \alpha_g \left[\underbrace{\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2}}_{\sim \frac{\Delta T}{t_b} \frac{\Delta T}{r^2}} \right].$$
(7)

Five factors affect the order of magnitude of these three groups. First, the characteristic temperature difference ΔT influences all groups equally, thus it does not affect their relative importance. Second, the heat injection/extraction characteristic time t_q affects the leftmost term which is related to the thermal inertia of the ground. Third, the residence time of the groundwater flow near the borehole, t_c , impacts on the remaining two terms on the left hand side of Equation (7), linked to heat convection in the ground. Fourth, the characteristic transversal diffusion time t_b affects the group at the right hand side of the equation that represents heat conduction in the ground. Although the relation between the characteristic times t_b , t_c , and t_q has already been discussed, the relative importance of the three groups in the governing equation also depends on the distance to the borehole divided by the borehole radius r/r_b .

Close to the borehole, at radial distances comparable to the borehole radius, $r/r_b \sim 1$, the sequence of characteristic times dictates the relative importance of each group. There, thermal inertia of the ground is negligible compared to heat conduction as $t_b \ll t_q$. Heat conduction also dominates over the convective transport of heat in the ground since, in the present work, the Peclet number of the groundwater flow is small compared to unity, $t_b/t_c \sim \text{Pe} \ll 1$. Hence, this inner region is, in first approximation, quasi-steady and solely governed by heat conduction.

Further away from the borehole, however, the described balance of terms changes giving rise to a second region. So, at distances from the borehole of order $r/r_b \sim \sqrt{t_q/t_b} \gg 1$, thermal inertia and heat conduction become equally important in the governing equation. The relative importance of heat convection in the outer region relies on the Peclet number. The present work focuses on the most interesting case in which all three phenomena (thermal inertia, heat convection, and

heat conduction) are important in the outer region for which $\text{Pe} \sim \sqrt{t_q/t_b} \ll 1$. Hence, this outer region is governed by an unsteady convection-conduction equation.

Matched asymptotic expansion techniques

The two-region structure described before is exploited using matched asymptotic expansion techniques (Lagerstrom 1988). These mathematical tools address the inner and outer regions separately to afterwards match their solutions at an intermediate region in which both solutions are valid. To tackle each of the two regions, asymptotic expansion techniques exploit the presence of small parameters in the problem by expressing the solutions to the inner and outer regions, T_{in} and T_{out} respectively, as polynomial expansions of that small parameter. In the present work, the Peclet number of the groundwater flow acts as the small parameter so that the expansions to use are

$$T_{\rm in} = T_{\rm in}^{(0)} + \operatorname{Pe} T_{\rm in}^{(1)} + \mathcal{O}\left(\operatorname{Pe}^2\right), \qquad T_{\rm out} = T_{\rm out}^{(0)} + \operatorname{Pe} T_{\rm out}^{(1)} + \mathcal{O}\left(\operatorname{Pe}^2\right), \qquad T_j = T_j^{(0)} + \operatorname{Pe} T_j^{(1)} + \mathcal{O}\left(\operatorname{Pe}^2\right). \tag{8}$$

Although logarithmic dependencies on the small parameter, of the form $\ln(\text{Pe})$, do appear as well in the asymptotic solution of the problem, these are treated as simple numbers instead and are incorporated as such into the zeroth order solution and first order correction to obtain (Rico and Hermanns 2024b). Substitution of all these expansions into the governing equations, continuity conditions, and boundary conditions of the inner and outer regions supplies a set of mathematical problems whose sequential solution delivers the different terms of the expansions.

Inner region

The inner region is composed by the grout filling up the borehole and by the ground located at distances of order $r \sim r_b$. Since $t_b \ll t_c \ll t_q$, the unsteady term of the governing equation in the grout is negligible compared to heat conduction in both the zeroth order solution $T_{\rm in}^{(0)}$ and the first order correction $T_{\rm in}^{(1)}$ of the inner problem.

The aforementioned sequence of characteristic times, $t_b \ll t_c \ll t_q$, also implies that, in first approximation, the ground located in the vicinity of the borehole is governed solely by heat conduction so that $T_{in}^{(0)}$ satisfies the same equation in ground and grout. However, the governing equation in the ground for $T_{in}^{(1)}$ slightly changes. While the right hand side of the equation remains equal in ground and grout, the forcing term that arises from substituting $T_{in}^{(0)}$ into the convective terms is now as relevant as the right hand side. Consequently, $T_{in}^{(1)}$ includes additional terms associated with the particular solution of the governing equation.

Apart from the governing equations, the asymptotic expansions must be substituted into the boundary conditions as well. However, the boundary condition at infinity cannot be fulfilled due to the presence of the outer region. This fact leads to the presence of undetermined integration constants in $T_{in}^{(0)}$ and $T_{in}^{(1)}$ that will be specified through an asymptotic matching with the outer solution.

Outer region

Far from the borehole, at distances of order $r \sim r_b \sqrt{t_q/t_b} \gg 1$, all three physical phenomena are relevant. Fortunately, the velocity field at such distances from the borehole behaves like an uniform stream so that

$$\mathbf{v}_r \Big|_{r \sim r_b \sqrt{t_q/t_b} \gg 1} = \cos(\theta) + O\left(\mathrm{Pe}^2\right) \qquad \text{and} \qquad \mathbf{v}_\theta \Big|_{r \sim r_b \sqrt{t_q/t_b} \gg 1} = -\sin(\theta) + O\left(\mathrm{Pe}^2\right). \tag{9}$$

This single simplification makes the partial differential equation in the outer region solvable. Furthermore, since the corrections in the velocity field are of order Pe², the very same equation governs both the zeroth order solution and the

first correction to the outer region.

This governing equation is solved along with the boundary condition at infinity. The remaining continuity and boundary conditions at the borehole wall and at the pipes, however, cannot be enforced due to the presence of the inner region. The asymptotic matching with the inner solution will determine the unspecified integration constants to fully obtain the solution to the outer region.

Matching of both regions

At an intermediate distance, located beyond the inner region but closer to the borehole than the outer region, both inner and outer solutions must be asymptotically equivalent. That is, their difference must be smaller than the order of the expansion:

$$\left(T_{\rm in}^{(0)} + \operatorname{Pe} T_{\rm in}^{(1)}\right)\Big|_{r \gg r_b} - \left(T_{\rm out}^{(0)} + \operatorname{Pe} T_{\rm out}^{(1)}\right)\Big|_{r \ll r_b\sqrt{t_q/t_b}} \ll \operatorname{Pe}.$$
(10)

This condition is satisfied through adequately choosing the yet-unspecified integration constants from the derived inner and outer solutions.

NUMERICAL EXAMPLES

The capabilities and limitations of the developed model are demonstrated in this section by comparing its results against detailed numerical simulations of the complete heat transfer problem using the commercial software package COMSOL (Comsol Inc. 2018). The borehole configuration selected for this example, depicted in Figure 2, is borrowed from Rico and Hermanns 2024b so that all details of the configuration can be found there.

Figure 3 shows the temporal evolution of the grout/ground temperature for the time-constant values of the heat injection rates per unit pipe length $q_1(t) = 18$ W/m and $q_2(t) = 54$ W/m and for a Peclet number equal to 0.03. This time evolution is depicted through three snapshots at different times, namely, 1 week at the top, 10 weeks at the middle, and 100 weeks at the bottom. Inner and outer solutions are depicted using solid black lines on the left and right plots in the figure, respectively, while the reference solution is represented in both plots using a solid color map.

Results in the inner region show the dependence of the developed asymptotic model on the assumption that $t_b \ll t_q$. For the not-so-large value of 1 week, Figure 3 shows relevant discrepancies between the inner and reference solutions as for that time the quasi-steady region is constricted.



Figure 2 Borehole configuration for the numerical examples.

As time evolves, though, the quasi-steady region expands and the accuracy of the inner region enhances, practically overlapping the reference solution for 100 weeks, when the final steady-state is essentially reached.

The excellent performance exhibited by the outer region in all snapshots shown in Figure 3 is explained by its mathematical formulation. In contrast to the inner region, in which the governing equations are heavily simplified, the governing equation in the outer region retains all three phenomena, namely, thermal inertia, heat convection, and heat conduction. It only differs from the full governing equation in terms related to the velocity field that decay quadratically with the distance to the borehole. Hence, far from the borehole the differences are minimal leading to the results shown in Figure 3.



Figure 3 (Left) inner solution and (right) outer solution for Pe = 0.03 at different times. The solid color map represents the reference solution, derived from detailed numerical simulations in COMSOL, while the black lines show temperature isolines computed using the proposed asymptotic model.

Another example of the good performance of the proposed model is illustrated in Figure 4. It shows the time evolution of the ground temperature on a circle located at 50 borehole radii from the borehole center. That is, at $r = 50 r_b$. For convenience, time evolution is expressed in months in this figure, where one month is considered 365.25/12 days. Left and right plots represent the time evolution of ground temperature for two different values of the Peclet number, namely, 0.01 and 0.1 respectively. The shown results confirm the little impact the Peclet number of the groundwater flow has onto the accuracy of the model as deviations from the reference solution are negligible from an engineering point of view.

However, Figure 4 shows the strong impact of the Peclet number on the thermal influence of the borehole. Particularly, it reveals that the intensity of the thermal influence grows with decreasing Peclet numbers. This is a consequence of the inherent inefficiency of heat conduction that leads to a build up of heat around the borehole. This build up takes place in all spatial directions, not only downwards of the borehole, leading to the shift in temperatures observed on the left plot.



Figure 4 Comparison between reference solution (black) and outer solution (blue) on a circle of radius $r = 50 r_b$ for (left) Pe = 0.01 and (right) Pe = 0.1 at different times.

CONCLUSION

The present work has developed, in a mathematically rigorous way, a physically-sound model for the thermal interaction of geothermal boreholes with creeping groundwater flows, characterized by small Peclet numbers of the groundwater flow. Additionally, heating and cooling needs of the building are assumed to vary slowly, a simplification that matches most operating conditions of real-world geothermal HVAC systems.

The performed scale analysis of the problem reveals the presence of two distinct regions whose existence is exploited using asymptotic expansion techniques. The resulting model for the thermal interaction of geothermal boreholes with creeping groundwater flows exhibits great performance when compared against detailed numerical simulations.

ACKNOWLEDGMENTS

Grant PID2021-128172OB-I00 funded by MCIN/AEI/10.13039/501100011033, Spain and by "ERDF A way of making Europe".

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